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Before the
FEDERAL COMMUNICATIONS COMMISSION
Washington, D.C. 20554

In the Matter of:	)	
	)	ET Docket No. 96-102
Amendment of the Commission's Rules to	)	RM-8648
Provide for Operation of Unlicensed NII	)	RM-8653
Devices in the 5 GHz Frequency Range	)	

WIRELESS INFORMATION NETWORKS FORUM
COMMENTS IN SUPPORT OF PETITION FOR RECONSIDERATION

The Wireless Information Networks Forum ("WINForum") hereby respectfully submits its comments in support of the Petition for Reconsideration filed by Hewlett-Packard Company ("HP") in the above-captioned proceeding.[1] HP requests that the Commission reconsider the power limit for the 5.15-5.25 GHz band adopted in the recent Report and Order allocating 300 MHz of new spectrum for Unlicensed National Information Infrastructure ("U-NII") devices.[2] The change proposed by HP would significantly increase the utility of the lowest U-NII band, would be consistent with international developments, and would not, as shown in WINForum's technical attachments, cause harmful interference to co-channel mobile satellite service ("MSS")

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[1] Petition for Reconsideration of Hewlett-Packard Company, ET Docket No. 96-102 (Mar. 3, 1997) ("HP Petition").

[2] Amendment of the Commission's Rules to Provide for Operation of Unlicensed NII Devices in the 5 GHz Frequency Range, ET Docket No. 96-102, RM-8648, RM-8653 (Jan. 9, 1997) ("*U-NII Order*"). See also 62 Fed. Reg. 4649 (Jan. 31, 1997). Under Section 1.429(d) of the Commission's Rules, petitions for reconsideration are therefore due on March 3, 1997.

licensees. WINForum accordingly urges the FCC to adopt a transmitter output power limit of 250 mW, with up to 6 dBi of antenna gain,[3] for the 5.15-5.25 GHz band on reconsideration.

As one of the original petitioners seeking 5 GHz spectrum for unlicensed multimedia products, WINForum strongly commended the Commission for its prompt action to adopt the *U-NII Order*. This order allocated three bands of 100 MHz each near 5 GHz and established minimal technical rules designed to ensure the broadest range of technologies and equipment and the optimal compatibility between U-NII devices and other spectrum users. While, in large part, WINForum strongly supported the Commission's order, WINForum's technical committees extensively studied the regulations and noted a number of areas where clarifications or minor modifications would be in the public interest. Accordingly, on March 3, 1997, WINForum filed a Petition for Reconsideration and Clarification proposing changes that it believes are consistent with the intent of the Commission's rules.[4]

Specifically, WINForum's Petition for Reconsideration and Clarification raised the following points:

- *First*, WINForum urged the Commission to clarify that operation across the lower and middle band boundary at 5.25 GHz is permissible.[5] As discussed in the petition, the spectrum in the lowest 100 MHz (5.15-5.25 GHz) is immediately adjacent to the middle band (5.25-5.35 GHz), but currently subject to more stringent power and

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[3] For gains over 6 dBi, as in the other bands, devices should apply a dB for dB reduction in transmitter output power to maintain an overall EIRP of 1 Watt. Consistent with its other proposals on reconsideration, *supra* at 3, the actual power limit would be stated as 11 dBm + 10 log(***B***) dBm, where ***B*** is the 26 dB bandwidth in megahertz, up to a maximum of 250 mW.

[4] Petition for Reconsideration and Clarification of the Wireless Information Networks Forum, ET Docket 96-102 (filed Mar. 3, 1997) ("WINForum Petition").

[5] WINForum Petition at 5.

operational requirements. Nonetheless, if the regulations for the lowest band are met, there appears to be no reason to constrain U-NII devices from operating using channels that cross the 5.25 GHz boundary.

- *Second*, WINForum believed the Commission should clarify and harmonize the out-of-band emissions limitations with the general "quiet band" limits of Section 15.209.[6] The petition suggested extending the specific Section 15.407 out-of-band emissions limits from 10 MHz to 20 MHz from the band edges, and then applying the Section 15.209 quiet band limits beyond that point. Since the filing of the petition, WINForum has continued its discussions with NTIA, as manager of the spectrum immediately above and below the U-NII bands, considering the potential ramifications of the proposed changes. WINForum will continue to work with NTIA on this issue and apprise the Commission if such discussions require modification of the positions stated in its petition for rulemaking.

- *Third*, WINForum argued that the Commission should eliminate the frequency stability requirements, since no band channelization has been adopted.[7] With both the existing and the proposed out-of-band emission limits, all adjacent channel protection requirements are already inherent in the rules and frequency stability rules are unnecessary.

- *Fourth*, WINForum urged the Commission to revise the output power and power spectral density rules to avoid disadvantaging broadband systems.[8] Under the current rules, systems with a bandwidth of less than 1 MHz could be deployed and utilize the same power allowed for systems of 1 MHz, gaining a signal-to-noise advantage. Instead, WINForum suggested that the total power be specified as X dBm + 10 log(B), where B is the 26 dB bandwidth in MHz and X is 4 dBm for the 5.15-5.25 GHz band,[9] X is 11 dBm for the 5.25-5.35 GHz band, and X is 17 dBm for the 5.725-5.825 GHz band. WINForum also suggested allowing 3 dB of tolerance in any given 1 MHz band, while maintaining the total power output as a function of bandwidth (as specified above), recognizing that most modulation envelopes are not "spectrally flat."

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[6] *Id.* at 5-7.

[7] *Id.* at 7.

[8] *Id.* at 7-9.

[9] As discussed *supra* at 5-6, the change proposed by HP would result in the low band power limit becoming identical to the middle band power limit, *i.e.*, 11 dBm + 10 log(B), up to a maximum of 250 mW.

- *Fifth*, WINForum believed the Commission should state the out-of-band emission limits relative to the inband power limits, including the required power reduction for systems with more than 6 dBi of antenna gain, rather than as relative to actual transmitted in-band power.[10] By stating the out-of-band emission limit as a function of the maximum permitted in-band power equipment designers will be able to utilize in-band power reduction to meet the out-of-band limitations. This added flexibility could reduce the cost of certain types of equipment for the band, but ultimately would not alter the magnitude of the power transmitted into adjacent bands.

- *Sixth*, WINForum requested that the Commission clarify and modify the rules to specify the total power output in a manner that accurately represents the interference potential of U-NII devices.[11] As detailed in the petition, symbol-to-symbol envelope variations due to modulation are unimportant from the perspective of interference potential. Accordingly, WINForum proposed a series of definitions and modifications that achieved consistency with the definitions and tests in the ANSI C63.17 measurement procedures for unlicensed personal communications devices. More importantly, these definitions account for the short term envelope variations without undermining the intent of the rules.

- *Seventh*, WINForum urged the Commission to clarify the definition and measurement of power spectral density and peak power spectral density.[12] As detailed extensively in two attachments to its petition,[13] experimental and theoretical research demonstrates that using peak measurement techniques will significantly overstate the interference potential of U-NII devices due to the inherent randomness (noise-like variation) of a wideband signal measured with a narrowband filter. WINForum therefore proposed definitions and measurement techniques that it believes accurately and correctly represent the interference potential of U-NII devices and retain consistency with the intent of the Commission's rules.

- *Eighth*, above and beyond the measurement issues described above, WINForum noted that special rule considerations are necessary for impulse transmission techniques.[14]

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[10] *Id.* at 9-10.

[11] *Id.* at 11-15.

[12] *Id.* at 15-19.

[13] *See* Padgett, Jay E., "Wideband Emissions Through a Narrowband Filter and the Implications on Measurement of Power Spectral Density Using a Spectrum Analyzer" (Mar. 3, 1997) (attached to WINForum Petition at Att. A); Johnson, Donald C., "Wideband Digitally Modulated Signals Emission Statistics and Measurement" (Mar. 3, 1997) (attached to WINForum Petition at Att. B).

[14] *Id.* at 20-21.

Because extremely wideband signals of short duration (*e.g.*, 10 ns) cannot accurately be measured with conventional spectrum analyzers due to limitations in the response time of the resolution filters, some special treatment under the rules is required if these devices are to be permitted. WINForum continues to work in conjunction with NTIA to develop a technique for accurately representing the interference potential of these devices.

- *Finally*, WINForum urged the Commission to modify the definition of U-NII devices to require such devices to utilize digital modulation techniques.[15] At present, the rules require digital communications, which WINForum believes was intended to foster the development of advanced wideband digital radio technologies. In furtherance of this intent, WINForum urged the Commission to modify section 15.403(a) to require U-NII devices to employ digital modulation.

WINForum believes that the rule clarifications and changes suggested in its petition will enhance the utility of the U-NII bands without altering the interference potential of U-NII devices. The proposed modifications thus are fully consistent with the Commission's intent, and adopting them would be in the public interest.

In addition to supporting the petition for reconsideration of WINForum, HP has requested reconsideration of the power limit adopted by the Commission for the low (*i.e.*, 5.15-5.25 GHz) band. Presently, the *U-NII Order* sets forth a power limit of 50 mW with up to 6 dBi of antenna gain and restricts low band U-NII devices to indoor-only operation. This translates into a total EIRP limit of 200 mW, five times less than the limit adopted for the adjacent 5.25-5.35 GHz band and five times less than the EIRP specified by ETSI for HIPERLAN devices. The *U-NII Order* states, however, that "if European HIPERLAN systems proliferate and operate at more power than U-NII devices," the Commission would consider changes to the domestic U-NII power limit in the low band.[16] As HP correctly observes, "as of October 1, 1995, twenty CEPT

[15] *Id.* at 21.

[16] *U-NII Order* at ¶ 96.

members had committed themselves to apply the terms of the European Radiocommunications Committee's HIPERLAN decision, including the ETSI standard for HIPERLAN devices."[17] As HP further notes, the ETSI standard contemplates operation at power levels up to 30 dBm, or 1 Watt EIRP.[18] Under the circumstances, WINForum agrees with HP that "[t]here is no reason for the Commission to defer authorizing U-NII devices at up to one watt of power."[19] Specifically, WINForum suggests, consistent with its proposal for the 5.25-5.35 GHz band in its petition for reconsideration, that the power limit for the lower band be 11 dBm + 10 log($\boldsymbol{B}$), where $\boldsymbol{B}$ is the bandwidth in megahertz, up to a maximum of 250 mW.[20]

In fact, independent of European developments, WINForum does not believe that U-NII devices operating at up to 250 mW transmitter output power[21] with 6 dBi of antenna gain will cause harmful interference to MSS feeder links operating in the 5.15-5.25 GHz band. As shown in the attached interference analyses,[22] harmful interference would not be caused even if a significant percentage of U-NII devices were operated outdoors. Because the current rules limit

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[17] HP Petition at 2.

[18] *Id.*

[19] *Id.*

[20] As discussed *infra* at 4, the power limits stated are for antenna gains of 6 dBi or less. As with other bands, any increase in antenna gain above 6 dBi would require a dB for dB reduction in transmitter output power.

[21] 11 dBm + 10 log($\boldsymbol{B}$) is roughly equivalent to 250 mW for a nominal bandwidth of 20 MHz. At 250 mW with 6 dBi of antenna gain, the total EIRP would be 1 Watt, consistent with HP's petition and the HIPERLAN specification.

[22] See Johnson, Donald C., "Average Antenna Gain for NII/SUPERNet Devices" (Dec. 3, 1996) (provided as Att. A); Padgett, Jay E., "Average Antenna Gain of Part 15 Devices As Seen By a Low Earth Orbit Satellite (Dec. 4, 1996) (provided as Att. B); "Effect of NII/SUPERNet Device Deployment on Globalstar™ Capacity" (Dec. 11, 1996) (provided as Att. C).

U-NII devices to indoor-only operation, the effect of authorizing U-NII devices at HP's requested power level will not cause harmful interference to MSS feeder links in the 5.15-5.25 GHz band.

WINForum urges the Commission to reconsider the U-NII Order and implement both the changes proposed by WINForum and the power limit increase requested by HP. Both WINForum and HP have requested modifications that are consistent with the Commission's twin goals of creating the most useful and diverse array of U-NII products and avoiding interference to co-channel spectrum users.

Respectfully submitted,

WIRELESS INFORMATION
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ATTACHMENT A

Average Antenna Gain for NII/SUPERNet Devices

Analysis of the effect of average NII/SUPERNet antenna gain to MSS feeder uplinks using the gain pattern of the AirTouch comments on ET Docket 96-102

Donald C. Johnson

Abstract

This paper shows that antenna gain of NII/SUPERNet devices has little effect on the mean signal level these devices will generate in the MSS feeder uplink band. In most cases this level is less with high gain antennas than with the same average power level and low gain antennas.

The signal level created by NII/SUPERNet devices at the MSS satellite is the result of a large number of device transmissions, thus if antennas with high gain are considered to be pointed in totally random directions the overall average gain is the same as if all antennas were omnidirectional. Any situation in which the average gain is higher than unity must result from some systematic pointing arrangement. This paper investigates the effect when all high gain antennas are pointed in a horizontal direction which is the only likely direction which may be favored.

The antenna gain template used in the AirTouch comments on the NII/SUPERNet docket (FCC ET Docket 96-102) is used.

The MSS satellite receiver antenna pattern is assumed to extend to a range at which the earth radio observer elevation angle to the satellite is 10 degrees. Depending on refraction conditions, this is shown to cover a radius of 1500 to 2200 miles from beneath the satellite. Thus, in some conditions, the satellite coverage includes the whole North American continent.

Device density patterns in terms of percent of devices at each vertical pointing angle are developed representing the worst case under each of 2 radio diffraction conditions and the overall average gain in the direction of the satellite is computed with each density pattern. These worst density patterns only occur at specific satellite locations. They represent the cases where most devices are at the maximum and most sensitive distance from the satellite.

At the worst satellite positions, the highest average gain of the horizontally pointing antennas is 1.0 dB (relative to isotropic) and occurs at a vertical beamwidth of 28° in the worst diffraction condition. In most density situations, a collection of antennas with gain greater than 1 systematically pointing in the horizontal direction creates an average gain less than 1 (0 dBi). Thus, limiting the antenna gain for NII/SUPERNet devices (by specifying a limit on EIRP rather than transmit power) will very likely result in higher mean signal level at the MSS satellite.

Outline

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Appendix 2. Example Computation of Average Gain using the Case1 and Case 2 Device Densities.

Appendix 3. Results of Average Gain Computation at Better Resolution for the Numerical Integration

Appendix 4. Some Information on Parabolic Antennas

1.0 Purpose

The appendix to the AirTouch reply comments on ET docket 96-102 (hereinafter referred to as the AirTouch analysis) contains an analysis of the average antenna gain of a large number of NII/SUPERNet devices in the direction of a low orbit satellite with the NII/SUPERNet devices configured with horizontally pointing directional antennas. This analysis looks at this situation in more detail to show the effect of high gain NII/SUPERNet antennas on the mean signal level at the MSS satellite.

The deployment assumptions, transmitter duty cycle and other arrangements of the AirTouch analysis are questionable, but are not challenged here; the scope of this paper is limited to the effect of antenna gain.

The satellite receiver has an iso - flux antenna pattern that provides equal attenuation to devices with an elevation angle toward the satellite from directly beneath (90 degrees) to as low as 10 degrees. The satellite altitude is 879 miles; at this altitude a 10 degree elevation angle indicates that the iso-flux pattern extends to about 1500 to 2200 miles from directly beneath the satellite. This means that a satellite over the north central region of the US would cover the whole continent in an iso -flux manner.

This analysis assumes the same relative NII/SUPERNet device antenna gain pattern as the AirTouch analysis and extends the analysis by evaluating some actual device density situations and providing more detail.

The conclusion is that the worst average gain is about 1 dB, relative to an isotropic antenna, and that in the typical situations, the average gain is less than 0 dB with high gain antennas.

2.0 The Antenna Pointing Arrangement

At the power levels proposed for NII/SUPERNet devices in the band shared with MSS it would require a very large number of devices transmitting simultaneously to cause a measurable level at the satellite. Thus, if all device antennas are arranged in totally random directions the average gain is unity regardless of the individual antenna gains. Only if the device antennas are systematically pointed in a direction toward the satellite will the average gain be greater than unity.

The only likely systematic pointing direction is horizontal at the location of the device. Since the vertical angle to the satellite is as low as 10 degrees at the most distant locations, devices with gain greater than unity pointing horizontally may generate average gains greater than unity in the direction of the satellite in some cases. This paper investigates those cases.

If the devices are used in an outside point-to-point link, their pointing direction will likely be horizontal. In this case, the gain of the antennas will likely be as high as practical and permitted by the rules. If the devices are used inside buildings (the principal intended use), access points or base stations with gain above 0 dBi may tend to point horizontally more than in other directions. However, in most cases the mobile pointing directions will be fully random. Thus, for inside devices the assumption that all antennas point in the horizontal direction represents an extreme worst case.

In sum, the pointing arrangement posited may apply to high gain outside point-to-point devices or to some portion of low and intermediate gain inside devices.

3.0 The Average Gain From the AirTouch Analysis.

The AirTouch analysis give an analytical expression for device antenna gain. This is the pattern template used here.

This pattern is a good representation for the purposes of defining average gain of distributed devices for the relative low gain that will normally be encountered in portable devices. However, the pattern does not cut off as sharply for higher gain antennas in the 5.2 GHz range as do most practical real antennas and is likely to overestimate the sidelobe power at high gain.

The analytical expression proposed represents a gain which averages more than 1 over a sphere and must be corrected by a factor in order to represent a real antenna. It is shown that with this correction, the average gain of the AirTouch analysis is always less than 1 dB.

The gain equation of the AirTouch analysis permits defining a horizontal and vertical beamwidth, but for purposes of calculating average gain it is observed that the average gain in any vertical direction with the template pattern is almost independent of the horizontal beamwidth (see following note). Thus, for the principal calculations here, the horizontal and vertical beamwidths are set equal.

From the AirTouch analysis, page 3 with $B_{w\varepsilon} = B_{w\alpha}$.

$$G_0 = \frac{27{,}000}{Bw_\varepsilon Bw_\alpha} \text{ and with the above assumption} \qquad (1)$$

$$Bw_\varepsilon = Bw_\alpha = Bw \qquad (2)$$

$$G_0 = \frac{27{,}000}{Bw^2}\text{. Define} \qquad (3)$$

$$M = 10^{\frac{-1}{2Bw^2}}\text{, then} \qquad (4)$$

$$G_a(\varepsilon,\alpha) = G_0 M^{\varepsilon^2} M^{\alpha^2} + 1 \qquad (5)$$

Equation 5 is the special case of the AirTouch gain expression when the horizontal and vertical beamwidths are equal. The general equation is:

$$G_a(\varepsilon,\alpha) = G_0 10^{-1/2(\varepsilon/B_{w\varepsilon})^2} 10^{-1/2(\alpha/B_{w\alpha})^2} + 1 \qquad (5a)$$

Note: That the average gain in any vertical direction is almost independent of horizontal beamwidth can be shown by inspecting equations 1 and 5a. From 5a, the horizontal beamwidth at any vertical angle is equal to $B_{w\alpha}$. Thus, the average of this term over all values of α is directly proportional to $B_{w\alpha}$. From 1, G_0 is inversely proportional to $B_{w\alpha}$. Thus, the product is independent of B_{wa} as is the complete expression 5a.

$G_a(\varepsilon, \alpha)$ is the gain of each individual antenna in the AirTouch analysis. However, the average of this gain is greater than unity, thus a correction is needed.

The AirTouch analysis evaluates an average gain toward the satellite for a collection of antennas pointing in evenly distributed random horizontal directions in accordance with the following expression.

$$G_{avg}(\varepsilon_1) = \frac{1}{2\pi(1 - Sin\varepsilon_1)} \int_{-\pi}^{\pi} \int_{\varepsilon_1}^{\pi/2} G(\varepsilon,\alpha) Cos\varepsilon \, d\varepsilon d\alpha \,, \qquad (6)$$

where ε_1 is the elevation angle from the most distant devices to the satellite and $G(\varepsilon, \alpha)$ is the gain of each antenna. The AirTouch equation uses G_a as the antenna gain. This is not used here because G_a has an average in excess of 1.

This is a correct expression for the gain of a collection of antennas at a given point in the sense that it is the ratio of the total power directed at an angle of ε_1 and above to the power that would be radiated in that direction by an ideal omni antenna. The AirTouch conclusion is that the maximum of this mean stated in deciBels is 2. However, some reflection will show that the value cannot be greater than 1 (0 dB) if the gain G is a real antenna gain in the sense that it averages 1 over the surface of a sphere. By the basic gain definition, a higher density of power flows in the direction $-\varepsilon_1$ to $+\varepsilon_1$ than flows at angles above or below ε_1. thus, the average gain above ε_1 must be less than 1.

Note: The expression is the average gain, of the random collection of antennas, in the direction of the partial spherical surface that consists of the surface above the angle ε_1 if the collection of antennas are considered to be located at a point. Thus, if the surface of this partial sphere contained an even density of emitters of gain G, all pointing horizontally, and the point collection of antennas are treated as a single omni directional receiver, the mean gain of the collection of emitters toward the receiver would be as given. This structure can then be inverted and the satellite can be considered the receiver. The iso-flux nature of the actual satellite receiver makes all devices appear to be at equal distance, thus the earth surface appears as the lower portion of a partial sphere relative to distance attenuation.

The expression would be correct if there were always the same number of devices within a solid angle of width $\delta\varepsilon\delta\alpha$ about the satellite and within the iso-flux pattern. This cannot be expected to be the case, however. For example, a satellite near the east coast might sense about all of the west coast devices at an elevation near $\varepsilon_1 = \pi/18$ (10 degrees) plus some devices in Canada, Mexico and South America (10 degrees is about 1500 to 2000 miles). However, the number of devices on the east coast within a circle (of much lower diameter) would only contain two small areas that include the dense east coast populated region. Thus, this satellite would experience a high density at low elevation angle and a low density at larger angles (shorter distance).

Further, if the density over the earth surface is constant, the number of devices in an angle of width $\Delta\varepsilon$ increases with decreasing angle. Thus, the number of devices in the angular width $\Delta\varepsilon$ is larger at longer distances.

G_{avg} is not the actual average gain, but a correction factor can be applied to it to make the average unity.

Define,

$$I_s(\varepsilon_1) = \int_{-\pi}^{\pi} \int_{\varepsilon_1}^{\pi/2} G_a(\varepsilon,\alpha) Cos\varepsilon \, d\varepsilon d\alpha \,. \qquad (7)$$

If the actual antenna gain = $k_1 G_a$ then the actual average antenna gain over the complete spherical surface is:

$$G_{avg}(-\pi/2) = \frac{k_1}{4\pi} I_s(-\pi/2) = 1$$

Then

$$k_1 = 4\pi \,/\, I_s(-\pi/2) \,, \qquad (8)$$

and with $G_{avg}(\varepsilon_1)$ = the actual average gain under the assumptions of the above note, then.

$$G_{avg}(\varepsilon_1) = \frac{k_1}{2\pi(1 - Sin\varepsilon_1)} I_s(\varepsilon_1) \qquad (9)$$

This average gain expression is only accurate for a particular device density distribution over the earth surface. It is the average gain of the antenna collection at angles above ε_1. If there are more devices at some angles than at others, this will affect the actual average gain.

Evaluation of I_s.

Equation 7 can be evaluated using numerical integration. This will be done next. The results can also be used to evaluate the average gain with other device density distributions to be investigated in a subsequent section.

Define $F(\alpha,\varepsilon)$ as follows:

$$I_s(\varepsilon_1) = \int_{-\pi}^{\pi} F(\alpha,\varepsilon_1) d\alpha$$

$$F(\alpha,\varepsilon_1) = \int_{\varepsilon_1}^{\pi/2} G_a(\varepsilon,\alpha) Cos\varepsilon \, d\varepsilon .$$

This can be further evaluated to

$$F(\alpha,\varepsilon_1) = G_0 M^{\alpha^2} \int_{\varepsilon_1}^{\pi/2} M^{\varepsilon^2} Cos\varepsilon \, d\varepsilon + 1 - Sin\varepsilon_1$$

$$I_1(\varepsilon_1) = \int_{\varepsilon_1}^{\pi/2} M^{\varepsilon^2} Cos\varepsilon \, d\varepsilon \qquad (10)$$

$$F(\alpha,\varepsilon_1) = G_0 M^{\alpha^2} I_1(\varepsilon_1) + 1 - Sin\varepsilon_1$$

$I_s(\varepsilon_1)$ can be further reduced to

$$I_s(\varepsilon_1) = G_0 I_1(\varepsilon_1) \int_{-\pi}^{\pi} M^{\alpha^2} d\alpha + 2\pi(1 - Sin\varepsilon_1).$$

Let

$$I_2 = \int_{-\pi}^{\pi} M^{\alpha^2} d\alpha \text{ , then} \qquad (11)$$

$$I_s(\varepsilon_1) = G_0 I_1(\varepsilon_1) I_2 + 2\pi(1 - Sin\varepsilon_1).$$

Then $I_1(\varepsilon_1)$ and I_2 can be evaluated by numeric integration.

The average gain at angles above ε_1 is given by the following:

$$G_{avg}(\varepsilon_1) = k_1 \left[1 + \frac{G_o I_1(\varepsilon_1) I_2}{2\pi(1 - Sin\varepsilon_1)} \right] \qquad (12)$$

4.0 The Average Gain Versus Elevation Angle

To understand the effect of actual potential device distributions it is instructive to investigate the average gain at specific elevation angles and then consider the actual anticipated device density. Call this average gain over all horizontal directions $G_e(\varepsilon)$.

Consider as before, a collection devices with antennas pointing in evenly distributed horizontal directions with gain G(α,ε), where α is the horizontal angle and ε is the vertical angle. The power flowing out of a small vertical angle $\Delta\varepsilon$ is

$$P_e(\varepsilon) = \frac{P_g}{4\pi r^2} \int_{\varepsilon_1 - \Delta\varepsilon/2}^{\varepsilon_1 + \Delta\varepsilon/2} \int_{-\pi}^{\pi} r^2 G(\alpha,\varepsilon) Cos\varepsilon d\alpha d\varepsilon = \frac{P_g}{4\pi} \int_{\varepsilon_1 - \Delta\varepsilon/2}^{\varepsilon_1 + \Delta\varepsilon/2} \int_{-\pi}^{\pi} G(\alpha,\varepsilon) Cos\varepsilon d\alpha d\varepsilon \; ,$$

where P_g is the power generated by the collection devices and r is the radial distance.

The integration over α will yield some function F(ε). Then, the integration of F(ε) over the differential limit range will yield F(ε)dε. Thus, $P_e(\varepsilon) = (P_g/4\pi)F(\varepsilon)d\varepsilon$. The ratio of the power through the differential angle with gain G to that with a gain of 1 is the power gain at the angle ε.

This power per unit elevation angle with a gain of 1 is easily shown to be $(P_g/2)Cos\varepsilon$.

Now consider a gain = $k_1 G_a(\alpha,\varepsilon)$.

From (5)

$$G_a(\varepsilon,\alpha) = G_0 M^{\varepsilon^2} M^{\alpha^2} + 1$$

Then F(ε) becomes

$$F(\varepsilon) = k_1 Cos\varepsilon \left[G_o M^{\varepsilon^2} \int_{-\pi}^{\pi} M^{\alpha^2} d\alpha + \int_{-\pi}^{\pi} d\alpha \right].$$

Then using (11)

$F(\varepsilon) = k_1 Cos\varepsilon \left[G_o M^{\varepsilon^2} I_2 + 2\pi \right]$, when the gain is $k_1 G_a(\alpha,\varepsilon)$.

The power density per unit elevation angle is $(P_g/4\pi)F(\varepsilon)$. This quantity divided by the power density with unity gain is the average gain over all horizontal directions at the angle ε. This was named $G_e(\varepsilon)$ above. This ratio is

$$G_e(\varepsilon) = k_1 \left[1 + G_o M^{\varepsilon^2} \left(\frac{I_2}{2\pi} \right) \right] \qquad (13)$$

5.0 Some Example Computations of Gain Versus Elevation Angle

The following equations were used in evaluating the integrals. The accuracy was checked with a Basic program with better resolution, however.

$$I_1(\varepsilon_1) \approx \Delta\varepsilon \sum_{n=\varepsilon_1/\Delta\varepsilon}^{n=(\pi/2\Delta\varepsilon)-1} M^{\left[\frac{180(n+.5)\Delta\varepsilon}{\pi}\right]^2} Cos[(n+.5)\Delta\varepsilon]$$

$$I_2 \approx 2\Delta\alpha \sum_{n=0}^{n=(\pi/\Delta\alpha)-1} M^{\left[\frac{180(n+.5)\Delta\alpha}{\pi}\right]^2}$$

The values of $\Delta\varepsilon$ and $\Delta\alpha$ were 0.0315 radians, corresponding to 1.803 degrees. This provides sufficient resolution to accurately reflect the gains at solid angles down to the 5 degree values shown in the following tables.

Note: The gain values were checked with a program written in MS Quick Basic and the accuracy was verified. This program was also used to verify the independence of the average gain with horizontal beamwidth.

Tables 1 through 4 show some example computations. Note that G_{avg} is less than 1 in all cases shown. However, the average gain at specific values of ε exceeds 1 at low elevation angles.

Table 1

B_w = 60 degrees $I_2(B_w)$ = 1.691 $G_o(B_w)$ = 7.500 $k_1(B_w)$ = 0.419 Maximum gain = 5.52 dB

ε_1 degrees	$I_1(\varepsilon_1)$	$I_s(\varepsilon_1)$	G_{avg} (ratio)	G_{avg} (dB)	$G_e(\varepsilon_1)$ (ratio)	$G_e(\varepsilon_1)$ (dB)
5	0.6078	13.62	0.9826	-0.08	1.26	1.00
10	0.5158	11.88	0.9471	-0.24	1.24	0.93
15	0.4569	10.58	0.9404	-0.27	1.21	0.82
20	0.3737	8.98	0.8990	-0.46	1.16	0.66
25	0.2982	7.50	0.8550	-0.68	1.11	0.46
30	0.232	6.15	0.8097	-0.92	1.05	0.23
35	0.1927	5.18	0.7998	-0.97	0.99	-0.04
40	0.1423	4.09	0.7541	-1.23	0.93	-0.33
45	0.1014	3.16	0.7097	-1.49	0.86	-0.64

Table 2

B_w = 30 degrees $I_2(B_w)$ = 0.865 $G_o(B_w)$ = 30.0 $k_1(B_w)$ = 0.373 Maximum gain = 10.6 dB

ε_1 degrees	$I_1(\varepsilon_1)$	$I_s(\varepsilon_1)$	G_{avg} (ratio)	G_{avg} (dB)	$G_e(\varepsilon_1)$ (ratio)	$G_e(\varepsilon_1)$ (dB)
5	0.345	14.69	0.954	-0.20	1.864	2.70
10	0.256	11.84	0.850	-0.71	1.727	2.37
15	0.203	9.92	0.794	-1.00	1.527	1.84
20	0.135	7.65	0.690	-1.61	1.296	1.12
25	0.085	5.82	0.598	-2.23	1.065	0.27
30	0.049	4.41	0.524	-2.81	0.860	-0.66
35	0.033	3.53	0.491	-3.09	0.694	-1.59
40	0.017	2.68	0.445	-3.52	0.572	-2.43
45	0.008	2.04	0.414	-3.83	0.488	-3.11

Table 3

B_w = 15 degrees $I_2(B_w)$ = 0.433 $G_o(B_w)$ = 120 $k_1(B_w)$ = 0.0.362 Maximum gain = 16.4 dB

ε_1 degrees	$I_1(\varepsilon_1)$	$I_s(\varepsilon_1)$	G_{avg} (ratio)	G_{avg} (dB)	$G_e(\varepsilon_1)$ (ratio)	$G_e(\varepsilon_1)$ (dB)
5	0.151	13.59	0.859	-0.66	3.00	4.77
10	0.075	9.11	0.636	-1.97	2.16	3.34
15	0.041	6.81	0.530	-2.76	1.31	1.17
20	0.014	4.84	0.424	-3.72	0.75	-1.26
25	0.003	3.80	0.380	-4.20	0.48	-3.15
30	0.00064	3.17	0.366	-4.36	0.39	-4.06
35	0.00018	2.69	0.364	-4.39	0.37	-4.34
40	0.00002	2.25	0.363	-4.41	0.36	-4.40
45	0.00000	1.84	0.362	-4.41	0.36	-4.41

Table 4

B_w = 7.5 degrees $I_2(B_w)$ = 0.216 $G_0(B_w)$ = 480 $k_1(B_w)$ = 0.360 Maximum gain = 22.4 dB

ε_1 degrees	$I_1(\varepsilon_1)$	$I_s(\varepsilon_1)$	G_{avg} (ratio)	G_{avg} (dB)	$G_e(\varepsilon_1)$ (ratio)	$G_e(\varepsilon_1)$ (dB)
5	0.0497	10.90	0.684	-1.65	3.92	5.94
10	0.0071	5.93	0.411	-3.86	1.13	0.52
15	0.001074	4.77	0.368	-4.34	0.42	-3.78
20	0.000025	4.14	0.360	-4.44	0.36	-4.42
25	1.84E-07	3.63	0.360	-4.44	0.36	-4.44
30	4.17E-10	3.14	0.360	-4.44	0.36	-4.44

6. 0 Overall Average Gain with Typical Device Density Distributions

G_{avg} as given by equation 6 is always less than 1 (0 dBi) when the antenna gain is corrected to average 1 over a complete sphere. This is verified in tables 1 through 4 above. However, equation 6 (used in the AirTouch analysis) represents the actual average gain only under a specific assumption of device density distribution.

Here two device distributions are considered in which there are the maximum number of devices at the longer ranges that will occur over the continental US, thus creating the worst condition for average gain for the particular maximum iso-flux range considered. It is assumed that the density of devices will form the same pattern as the population density of the continental US.

In case 1, the radial distance from beneath the satellite to the point where the local elevation angle to the satellite is 10 degrees is 2200 miles. Appendix 1 shows that this will be the approximate distance with relatively high atmospheric refraction.

In case 2, this distance is considered to be about 1500 miles, which corresponds to very low atmospheric diffraction.

<u>Case 1:</u>

The radial distance covered is 2200 miles. At this distance, the full east coast of the US has approximately a 10 degree elevation angle to a satellite over southeastern British Columbia, Canada, which is the worst case satellite location. It is estimated from an atlas that approximately 27% of the population of the US lives within about 200 miles of the east coast, and 200 miles covers an elevation angle between 10 degrees and 15 degrees with reference to the satellite position. All of Florida is outside the 10 degree angle, but the population of Florida is included in the estimate.

The population density per unit area (and corresponding device density) at distances corresponding to an elevation angle greater than 15 degrees was considered constant. The dense west coast population is at an angle greater than about 25 degrees, but the area covered by an arc through Los Angeles contains much of the low population density of the great plains.

The assumption is that the east coast population density increases linearly from the mean of the rest of the country at 15 degrees to a maximum at 10 degrees near the coast

This seems to be the worst case location for a satellite with a 2200 mile range.

<u>Case 2:</u>

For a satellite range of about 1500 miles radial distance to 10 degrees, it is possible to position a satellite so that both densely populated coasts appear at about the worst case elevation angle of 10 degrees. The satellite position for this is over eastern North Dakota. In this case, approximately 41% of the US population lives within the distance range corresponding to 10 degrees to 15 degrees. The density profile used includes the Florida population, although Florida is beyond the 10 degree angle range. In case 2 also, the density is considered to be distributed evenly at ranges corresponding to elevation angles above 15 degrees.

This can be considered the worst case device density distribution.

A satellite at the position assumed will move away from the most sensitive location very quickly. The satellite speed, relative to the earth surface is about 220 miles/minute. Thus, if it is moving in a southerly direction it will move to a position in which the elevation angles to both densely populated coasts become greater than 15 degrees within about 1 minute.

In most cases the device density when the satellite coverage includes the whole continental US will be more favorable than either of the above cases. That is, there will be a lower density at the longer range of the iso-flux pattern.

Appendix 2 shows the device density distribution for both cases and includes a table showing the computation of the average gain in each case with a vertical antenna beamwidth of 28 degrees. This was determined to be the worst case beamwidth for the case 2 device density distribution, as shown in table 5 below.

Table 5. Summary of Average Antenna Gain with Case 1 and Case 2 Device Density

Beamwidth (degrees)	Maximum gain with $B_{we} = B_{wa}$ (dB)	Case 1 Average gain (dB)	Case 2 Average gain (dB)
5	25.9	-4.32	-4.30
10	19.9	-1.73	-1.16
15	16.4	-.39	0.36
20	14.0	0.12	0.86
25	12.1	0.38	1.01
27	11.5	0.44	1.03
28	11.2	0.47	**1.04**
29	10.9	0.50	1.04
30	10.6	0.52	1.03
35	**9.4**	**0.58**	0.99
40	**8.38**	**0.58**	0.91
45	7.52	0.56	0.82
50	6.79	0.51	0.72
55	6.17	0.47	0.64
60	5.64	0.42	0.56
65	5.18	0.38	0.49
70	4.79	0.33	0.42

In both cases, if the antenna horizontal and vertical beamwidths are equal, the average gain is negative for antenna gains exceeding 20 dBi. In point-to-point applications the gain will almost always exceed 20 dBi. Thus, limiting the antenna gain will actually increase the mean signal level in this application.

The maximum average gain is about 1.0 dB and this occurs at a vertical beamwidth of 28 degrees and an antenna gain of about 11 dB. This occurs at the worst case device distribution and a satellite will see its effect only in extraordinary circumstances (negative refraction conditions) and for only about 1 minute on a specific orbit. Case 1 is a more typical worst distribution for normal diffraction conditions and the maximum average gain is about 0.6 dB in this case.

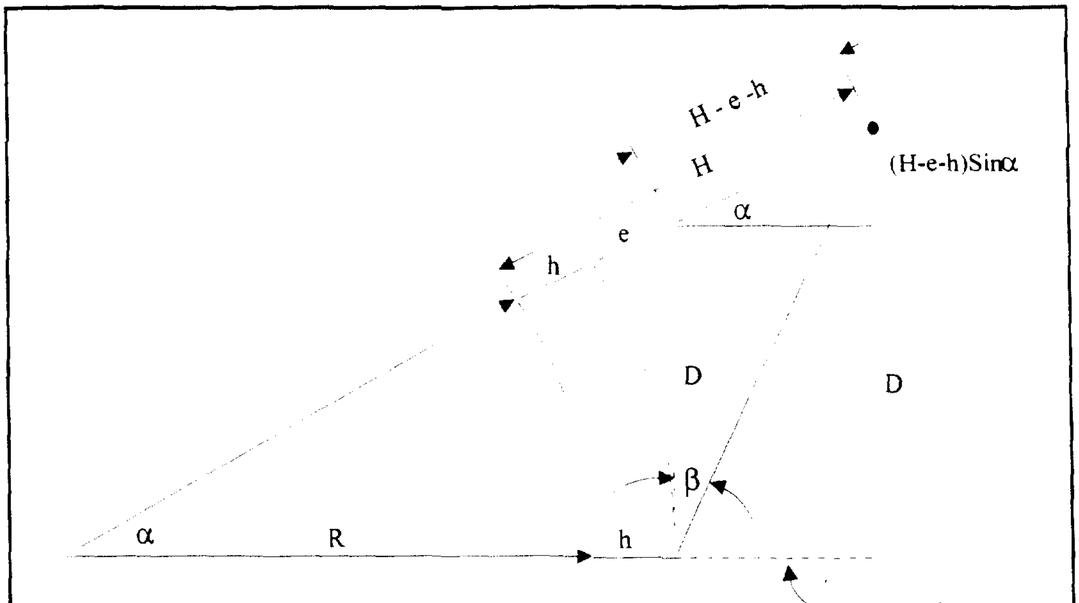
7.0 Conclusions

1. It is shown that the maximum average NII/SUPERNet device antenna gain for horizontally pointing antennas is 1 dB with the gain template used. This template can be considered to represent the actual possible antenna patterns for relatively low gain cases.

2. The worst case average gain for antennas with gains in excess of about 20 dBi is negative. This would mean that limiting the antenna gain will increase the satellite mean signal level level for point-to-point applications.

3. The average gain for fully random pointing antennas is 1 regardless of the individual antenna gains. The systematic horizontally pointing arrangement is not likely for indoor applications, so even the 1 dB average gain will not be realized in this case. The pointing arrangement over represents the actual case for overall situations.

Appendix 1. Computation of Elevation Angle vs. Distance



$$\text{Tan } \beta = \frac{(H\text{-}e\text{-}h)\text{Cos } \alpha}{D + (H\text{-}e\text{-}h)\text{Sin}\alpha}$$

Figure Ax. Antenna Pointing Angle
R = Earth Radius Times Diffraction Factor
D = Horizontal Distance to Device
h = Height of Device

This illustrates the geometry for computing the elevation angle between a device and the satellite. The device reference elevation angle pointing at the satellite is β. The angle from the satellite to the device relative to vertical at the satellite location is 90° - α - β.

The value of the earth radius is set at 4/3 (the diffraction factor) times actual to account for normal diffraction bending of the beam. R including the 4/3 factor is 5280 miles.

The value of e is:

$$e = (R + h)(Sec\alpha - 1)$$

The height of the device (h) is effectively zero relative to the satellite height and is set to zero in the computations that follow.

Appendix 1 Continued. Computations for 4/3 Earth Radius Multiplication Factor.

F = Earth radius multiplying factor for diffraction bending = 1.333

MSS Angle versus area covered — All distances in miles

H = Height of satellite = 878.6 miles
R = Earth radius 3960 miles
C = Earth distance to observer (over curved earth)
D = Horizontal distance to observer — F= **1.333**
e = Vertical distance observer to horizontal line from under satellite — e = (R+H)(SecAlpha-1)
Alpha = earth angle, observer to satellite — Alpha = Atan D/(R*F)
Angle at earth center between radial lines to two surface points
Beta = observer elevation angle to satellite
A = Area covered by radius D

Beta approximately = Atan[(H-e)CosAlpha/{D+(H-e)SinAlpha}]
A = (A/Pi)^(1/2)

A (sq miles)	C (miles)	Alpha (degrees)	e (miles)	D (miles)	Beta (degrees)	Sat angle from vertical
4.91E+06	1250.0	13.6	176.7	1280	25.29	51.15
5.11E+06	1275.0	13.8	184.0	1307	24.61	51.56
5.31E+06	1300.0	14.1	191.5	1334	23.94	51.95
5.52E+06	1325.0	14.4	199.1	1361	23.29	52.34
5.73E+06	1350.0	14.6	206.9	1388	22.65	52.70
5.94E+06	1375.0	14.9	214.9	1415	22.02	53.06
6.16E+06	1400.0	15.2	223.0	1442	21.41	53.40
6.38E+06	1425.0	15.5	231.3	1469	20.80	53.73
6.61E+06	1450.0	15.7	239.8	1497	20.21	54.05
6.83E+06	1475.0	16.0	248.4	1524	19.63	54.36
7.07E+06	1500.0	16.3	257.2	1552	19.06	54.66
7.31E+06	1525.0	16.5	266.1	1580	18.51	54.95
7.55E+06	1550.0	16.8	275.2	1607	17.96	55.22
7.79E+06	1575.0	17.1	284.5	1635	17.42	55.49
8.04E+06	1600.0	17.4	294.0	1663	16.89	55.75
8.30E+06	1625.0	17.6	303.6	1691	16.37	56.00
8.55E+06	1650.0	17.9	313.5	1720	15.86	56.24
8.81E+06	1675.0	18.2	323.4	1748	15.35	56.47
9.08E+06	1700.0	18.4	333.6	1776	14.86	56.69
9.35E+06	1725.0	18.7	344.0	1805	14.37	56.91
9.62E+06	1750.0	19.0	354.5	1833	13.89	57.12
9.90E+06	1775.0	19.3	365.2	1862	13.42	57.32
1.02E+07	1800.0	19.5	376.1	1891	12.95	57.51
1.05E+07	1825.0	19.8	387.1	1920	12.50	57.70
1.08E+07	1850.0	20.1	398.4	1949	12.04	57.88
1.10E+07	1875.0	20.3	409.8	1978	11.60	58.05
1.13E+07	1900.0	20.6	421.5	2008	11.16	58.22
1.16E+07	1925.0	20.9	433.3	2037	10.73	58.38
1.19E+07	**1950.0**	21.2	445.3	2067	**10.30**	58.54
1.23E+07	1975.0	21.4	457.5	2096	9.88	58.69

Appendix 1 Continued. Computations for Earth Radius Multiplication Factor of 1 (No Diffraction).

F = Earth radius multiplying factor for diffraction bending = 1.000

MSS Angle versus area covered — All distances in miles

H = Height of satellite = 878.62 miles — F = Earth radius multiplying fact

R = Earth radius 3960 miles — for diffraction bending =

C = Earth distance to observer (over curved earth)

D = Horizontal distance to observer — **1.000**

e = Vertical distance observer to horizontal line from under satellite — e = (R+H)(SecAlpha-1)

Alpha = earth angle, observer to satellite — Alpha = Atan D/(R*F)

Angle at earth center between radial lines to two surface points

Beta = observer elevation angle to satellite

A = Area covered by radius D

Beta approximately = Atan[(H-e)CosAlpha/{D+(H-e)SinAlpha}]

A = (A/Pi)^(1/2)

A (sq miles)	C (miles)	Alpha (degrees)	e (miles)	D (miles)	Beta (degrees)	Sat angle from vertical
3.14E+06	1000.0	14.5	158.5	1029	29.98	45.56
3.30E+06	1025.0	14.8	166.7	1056	29.06	46.11
3.46E+06	1050.0	15.2	175.2	1084	28.16	46.65
3.63E+06	1075.0	15.6	183.9	1111	27.29	47.16
3.80E+06	1100.0	15.9	192.9	1139	26.43	47.66
3.98E+06	1125.0	16.3	202.0	1167	25.59	48.13
4.15E+06	1150.0	16.6	211.5	1194	24.77	48.59
4.34E+06	1175.0	17.0	221.1	1222	23.96	49.04
4.52E+06	1200.0	17.4	231.0	1251	23.18	49.46
4.71E+06	1225.0	17.7	241.1	1279	22.40	49.87
4.91E+06	1250.0	18.1	251.5	1307	21.65	50.27
5.11E+06	1275.0	18.4	262.1	1336	20.91	50.65
5.31E+06	1300.0	18.8	273.0	1365	20.18	51.01
5.52E+06	1325.0	19.2	284.1	1394	19.46	51.36
5.73E+06	1350.0	19.5	295.5	1423	18.76	51.70
5.94E+06	1375.0	19.9	307.1	1452	18.08	52.03
6.16E+06	1400.0	20.3	319.0	1481	17.40	52.34
6.38E+06	1425.0	20.6	331.1	1511	16.74	52.64
6.61E+06	1450.0	21.0	343.5	1541	16.09	52.93
6.83E+06	1475.0	21.3	356.2	1571	15.45	53.21
7.07E+06	1500.0	21.7	369.2	1601	14.82	53.48
7.31E+06	1525.0	22.1	382.4	1631	14.20	53.74
7.55E+06	1550.0	22.4	395.9	1662	13.59	53.98
7.79E+06	1575.0	22.8	409.7	1692	12.99	54.22
8.04E+06	1600.0	23.1	423.7	1723	12.40	54.45
8.30E+06	1625.0	23.5	438.1	1755	11.82	54.67
8.55E+06	1650.0	23.9	452.7	1786	11.25	54.88
8.81E+06	1675.0	24.2	467.6	1817	10.69	55.08
9.08E+06	**1700.0**	24.6	482.9	1849	**10.13**	55.27
9.35E+06	1725.0	25.0	498.4	1881	9.58	55.46

With normal diffraction (F = 1.33), the distance at which the observer elevation angle (β) is 10 degrees is about 1950 miles. With no diffraction (F = 1.00) this range is 1750 miles. At either of these distances, the satellite will not be in range of the southern part of the US when the densely populated areas of the coasts are at the sensitive low angle.

The worst case device density (case 2) was based on a range at 10 degrees of 1500 miles. In this case, most of both coasts are at the 10 degree angle when the satellite is over eastern North Dakota. This range is only achieved in an abnormally high diffraction condition.

The case 1 range to 10 degrees was 2200 miles. At this range the complete east coast, including most of Florida, is approximately at the low elevation angle of 10 degrees and the satellite is over British Columbia, Canada. Negative diffraction is necessary to achieve this condition. Distances between those used will show less device density at the low end of the range of angle than will case 2.

Appendix 2. Example Computation of Average Gain using the Case1 and Case 2 Device Densities.

Case 1 device density: Low diffraction.

Diffraction multiplying factor	1.7
Distance to point of 10° elevation	2200 miles
Satellite location (worst case)	SE British Columbia

East coast of US at 10° elevation angle. 27% of devices within 200 miles of the east coast.

Case 2 device density: Normal diffraction.

Diffraction multiplying factor =	0.9
Distance to point of 10° elevation =	1500 miles
Satellite location	East Central N. Dakota

Both US coasts at 10° elevation angle. 41% of devices within 200 miles of the US coasts.

Antenna beamwidth (Bw) = 28°

		Case 1, 2200 mi radius Over BC, Canada		Case 2, 1500 mi radius Over East ND	
Angle	Ge	Density	Product		Product
10.00	1.788	7.94%	0.1419	10.58%	0.1891
11	1.745	7.07%	0.1233	8.94%	0.1560
12	1.699	6.04%	0.1026	7.43%	0.1263
13	1.652	4.87%	0.0805	6.04%	0.0997
14	1.602	3.60%	0.0576	4.76%	0.0762
15	1.550	2.23%	0.0345	3.59%	0.0557
16	1.498	2.15%	0.0322	3.41%	0.0511
17	1.445	2.08%	0.0300	3.23%	0.0467
18	1.391	2.00%	0.0279	3.06%	0.0425
19	1.337	1.93%	0.0258	2.89%	0.0386
20	1.283	1.86%	0.0238	2.73%	0.0350
21	1.230	1.92%	0.0236	2.58%	0.0318
22	1.177	1.97%	0.0232	2.44%	0.0287
23	1.126	2.01%	0.0226	2.30%	0.0259
24	1.075	2.04%	0.0219	2.17%	0.0234
25	1.027	2.06%	0.0212	2.04%	0.0210
26	0.979	1.97%	0.0193	1.93%	0.0189
27	0.934	1.89%	0.0176	1.81%	0.0169
28	0.890	1.80%	0.0160	1.71%	0.0152
29	0.848	1.72%	0.0146	1.60%	0.0136
30	0.809	1.63%	0.0132	1.50%	0.0121
31	0.771	1.63%	0.0126	1.40%	0.0108
32	0.736	1.63%	0.0120	1.31%	0.0097
33	0.703	1.63%	0.0115	1.23%	0.0086
34	0.672	1.63%	0.0109	1.14%	0.0077
35	0.643	1.59%	0.0102	1.06%	0.0068
36	0.616	1.44%	0.0089	0.99%	0.0061
>=37	0.591	29.67%	0.1753	16.12%	0.0953
Sum		100.00%	1.11		1.27
dB			0.47		1.04

Note that under typical diffraction conditions (about 2000 miles range to 10 degrees, appendix 1) when the satellite coverage covers the whole US continent, the minimum vertical angle will exceed the 10 degree assumed above. A good estimate is that if this minimum angle is greater than about 20 degrees, the overall average gain in the above case is less than 0 dB.

Appendix 3. Results of Average Gain Computation at Better Resolution for the Numerical Integration

Numerical integration at 0.1 degree increments.

The minimum vertical angle (Eps1 was)	10	degrees
The beamwidth vertical/horizontal ratio is	1	

Ge versus Epsilon at 60 and 10 degree beamwidths

Eps (degrees)	Ge (dB) at 60 degree Bw	Ge (dB) at 10 degree Bw
10	.927069	2.482751
11	.9077967	1.667826
12	.8867391	.829393
13	.8639103	-.0096
14	.8393259	-.8226725
15	.8130018	-1.581379
16	.7849562	-2.259451
17	.7552071	-2.837069
18	.7237766	-3.304688
19	.6906847	-3.664286
20	.6559554	-3.927459
21	.6196133	-4.111423
22	.5816841	-4.234791
23	.5421944	-4.314505
24	.5011736	-4.364317
25	.4586517	-4.394509
26	.4146606	-4.412298
27	.3692327	-4.422501
28	.3224028	-4.428205
29	.274207	-4.431314
30	.2246829	-4.432967
31	.1738694	-4.433825
32	.1218075	-4.43426
33	.068	-4.434475
34	.0141	-4.434579
35	-.0414	-4.434628
36	-.0980	-4.43465
37	-.1557086	-4.43466
38	-.2143265	-4.434665
39	-.2738672	-4.434667
40	-.3342778	-4.434668
41	-.3955051	-4.434668
42	-.4574935	-4.434668
43	-.5201863	-4.434669
44	-.583526	-4.434669
45	-.6474535	-4.434669
46	-.711909	-4.434669
47	-.7768312	-4.434669
48	-.8421582	-4.434669
49	-.9078266	-4.434669
50	-.973773	-4.434669

Note: The above is not intended to imply 6 digit accuracy. The accuracy is about 4 digits.